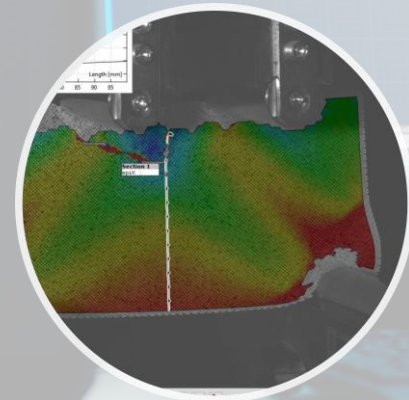
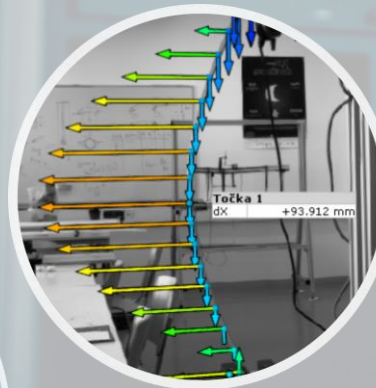
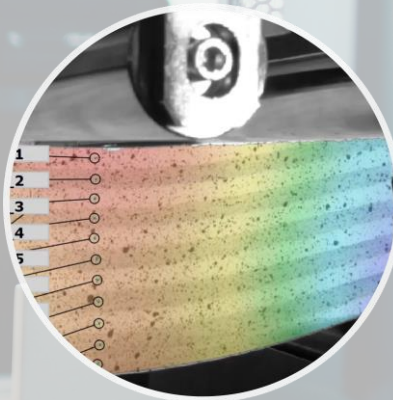


MINOR IN:

# APPLIED MECHANICS

## IN MODERN ENGINEERING PRACTICE



**G**  
**F**

Sveučilište  
u Rijeci  
Građevinski  
fakultet

MINOR IN:

# APPLIED MECHANICS

## IN MODERN ENGINEERING PRACTICE

Can be enrolled as part of the Graduate study programme in **Civil Engineering** module **Structures**

**4 optional courses** in the 2. semester

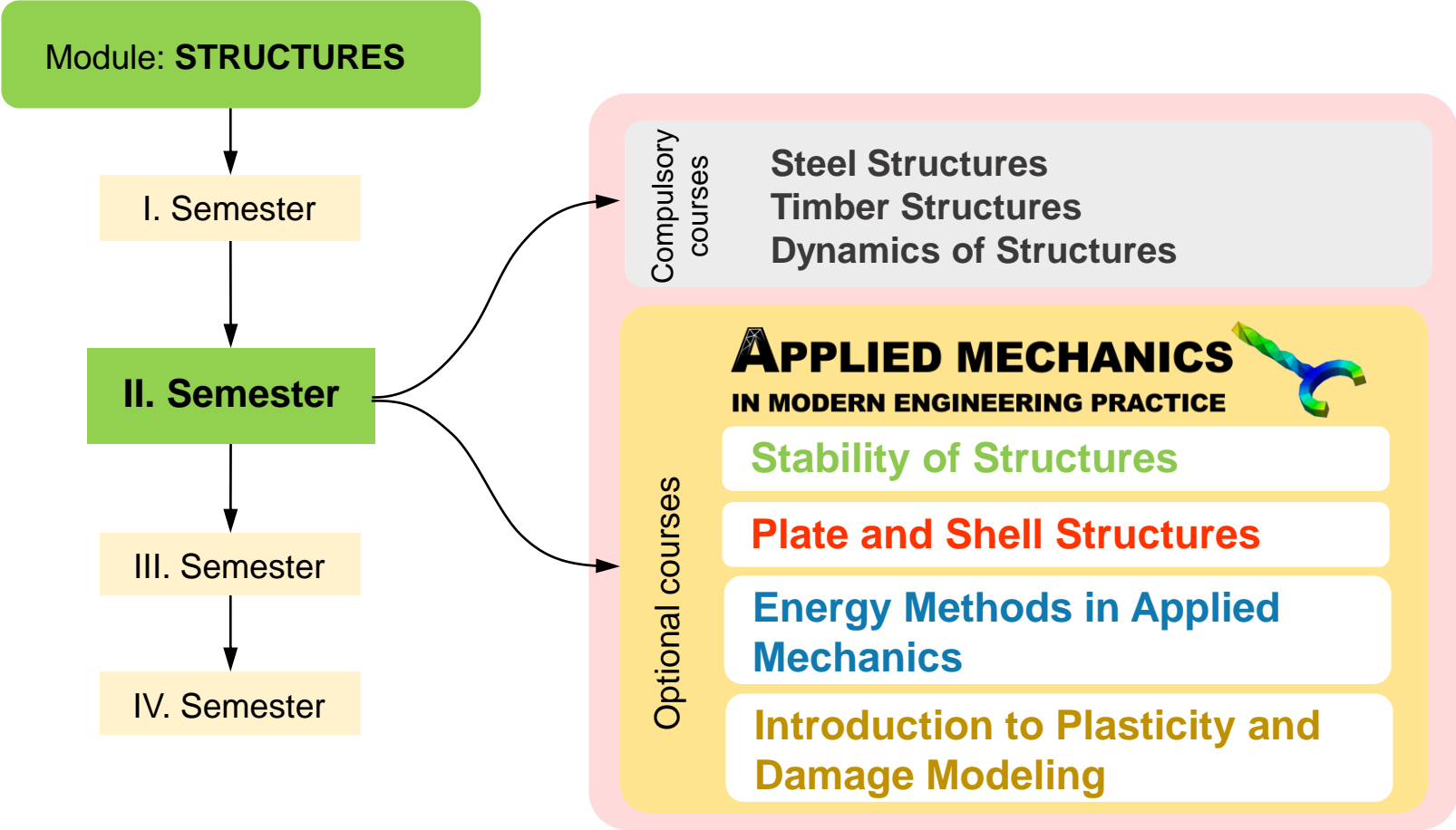
Can be enrolled as a **life-long education programme** (available to non-students)

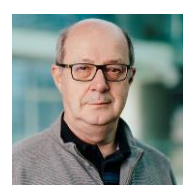
Carried out in **English language**

Improved **knowledge, skills and competences** + new **employment opportunities**

Certificate of an acquired minor degree in **Applied mechanics in modern engineering practice**

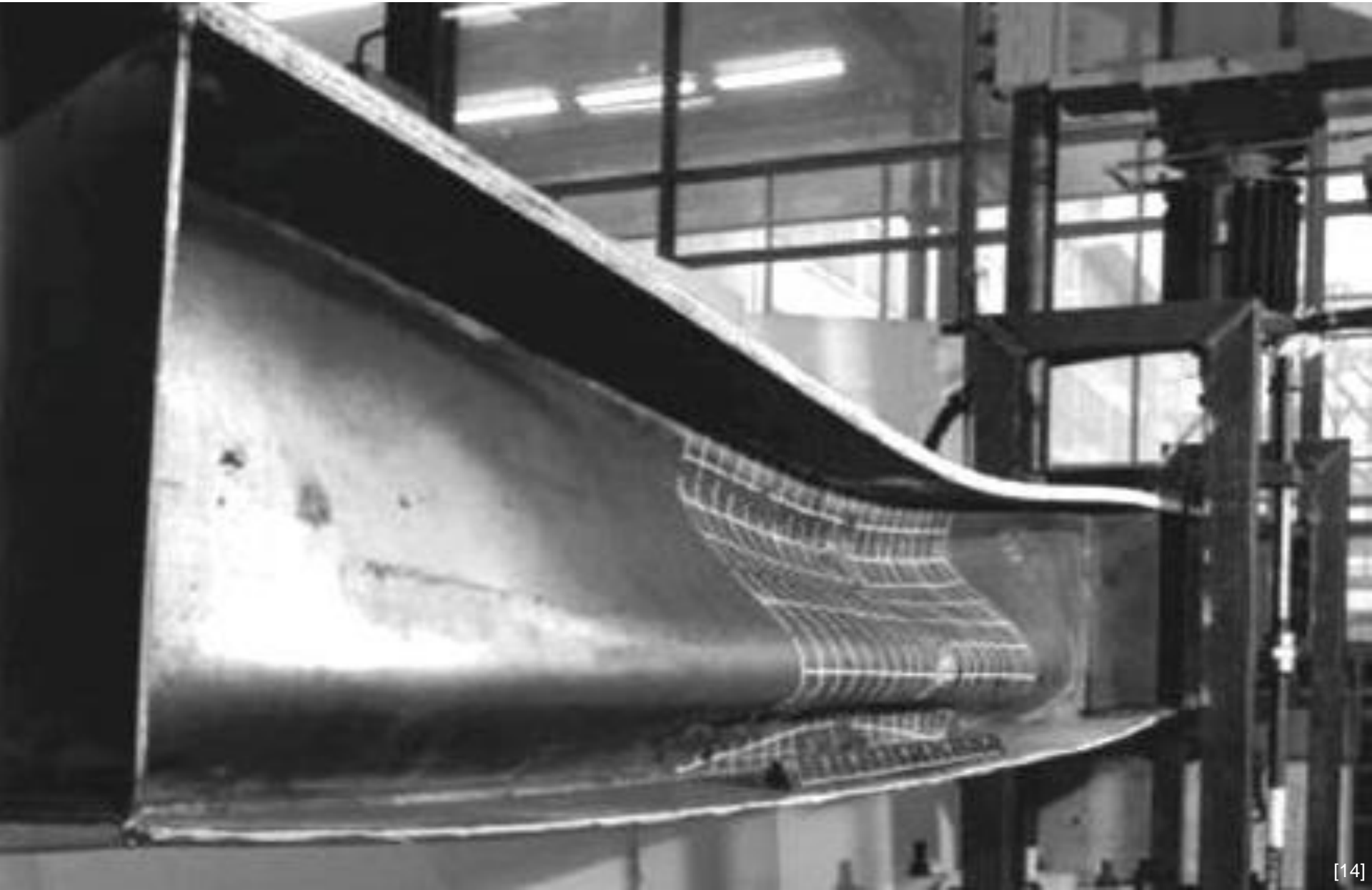
# Academic graduate programme in CIVIL ENGINEERING



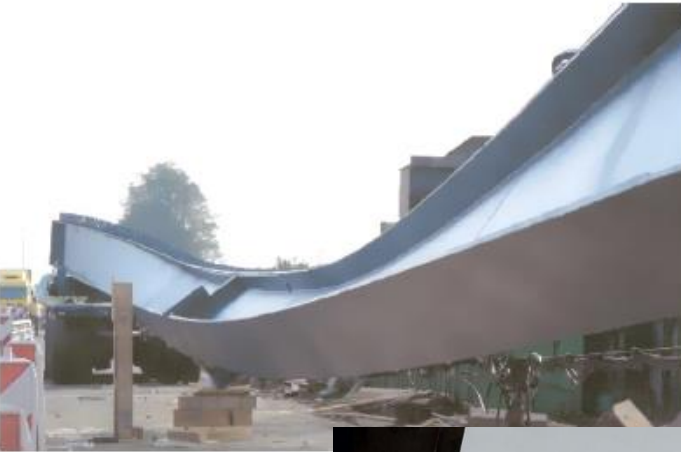


Assoc. Prof. **Dragan Ribarić**  
[dragan.ribaric@uniri.hr](mailto:dragan.ribaric@uniri.hr)  
G-328

# Stability of Structures



# Stability of Structures



[15]



[16]

[17]



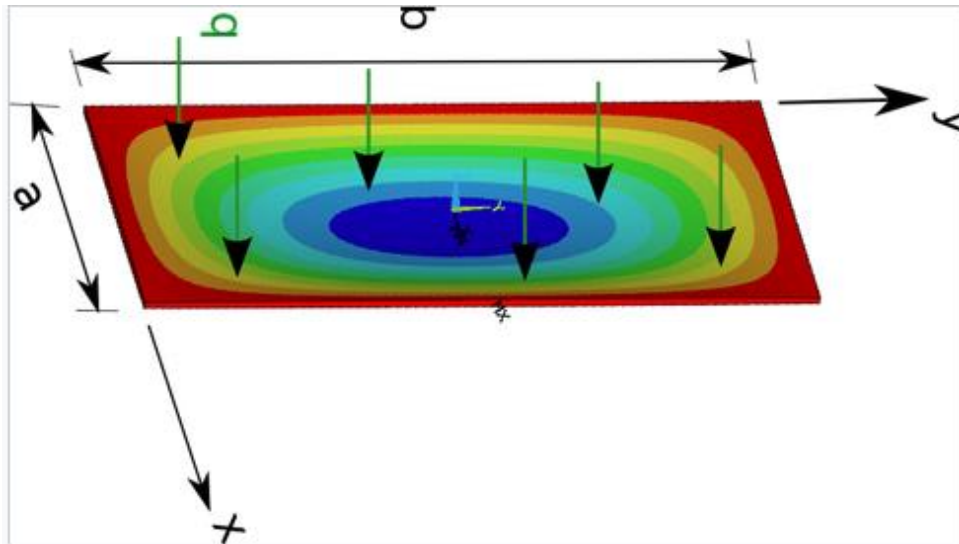
There is practical need to analyse safety against the instabilities caused by **structural geometry and internal forces combined**.

The new technical discipline arises for that purpose – **stability of structures**.

# Stability of Structures

## Theory

The problem of the geometrical instability is expressed by unified differential equations as a result of combined kinematic and material expressions as well as equilibrium relations.



$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \left( \frac{n_x}{D} \frac{\partial^2 w}{\partial x^2} + 2 \frac{\tau_x}{D} \frac{\partial^2 w}{\partial x \cdot \partial y} + \frac{n_y}{D} \frac{d^2 w}{dy^2} \right) = \frac{q}{D}$$

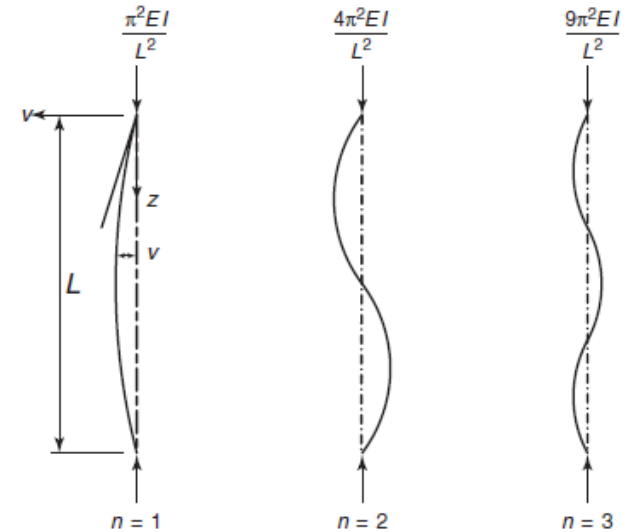


Fig. 2.5 Mode shapes of buckled column.

[19]

$$\frac{d^4 w}{dx^4} + \frac{P}{EI} \frac{d^2 w}{dx^2} = \frac{q}{EI}$$

**Leonhard Euler**  
(1707 – 1783)  
mathematician

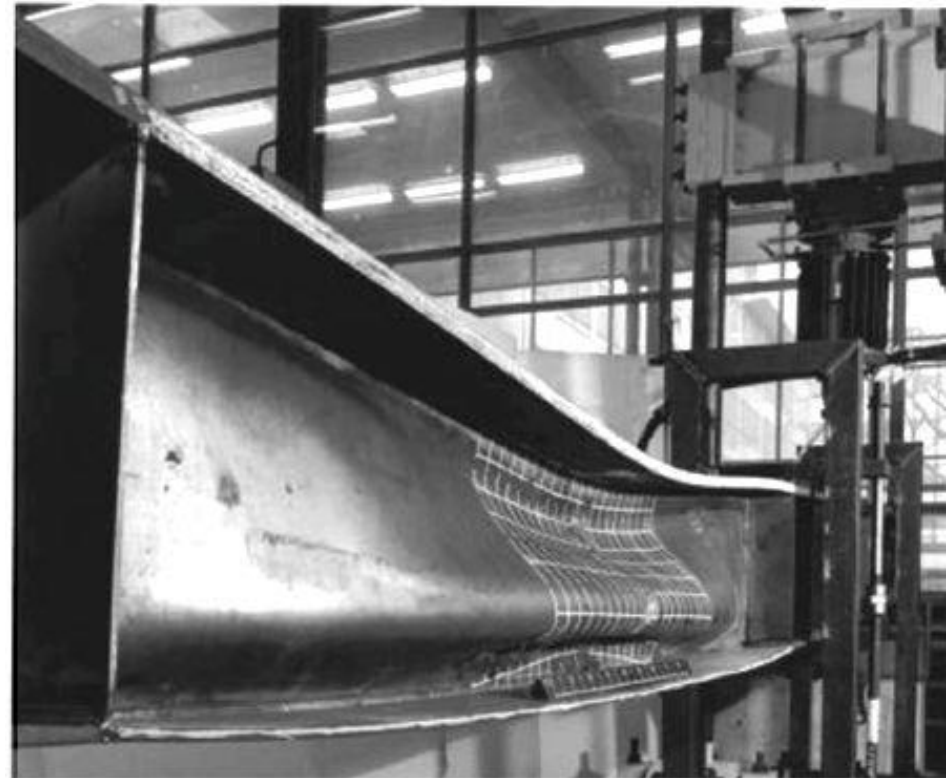


[20]

# Stability of Structures

## Experiments

Experimental setup models the practical problem and confirms the theoretical results.



[14]

Figure 2. Vertical web buckling.

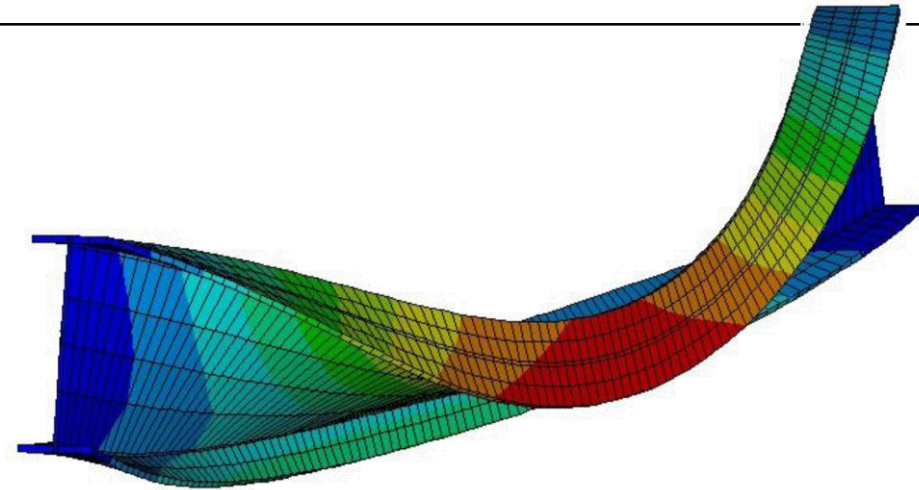
[21]

# Stability of Structures

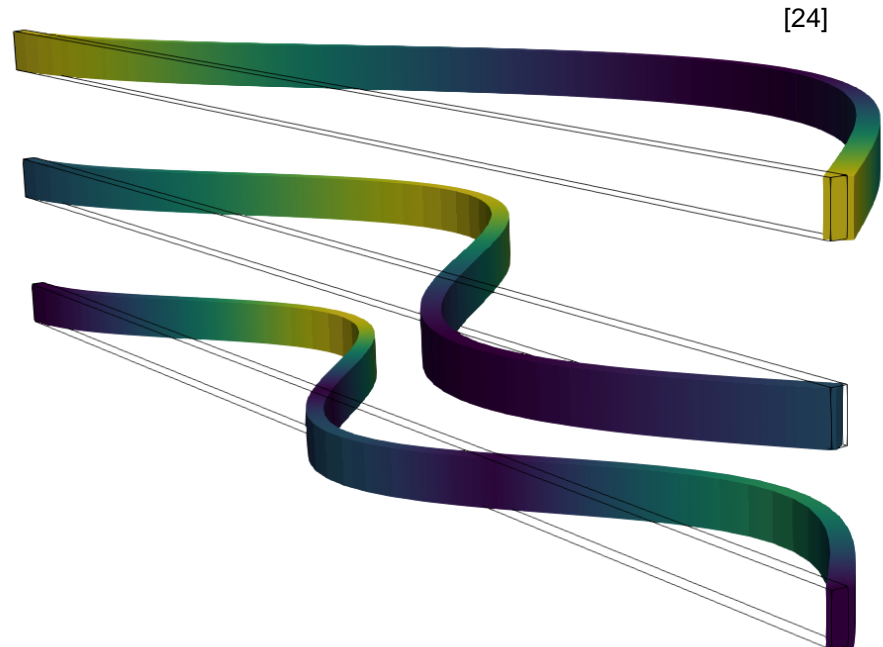
## Numerical solutions

Complex practical problems can be solved with numerical methods.

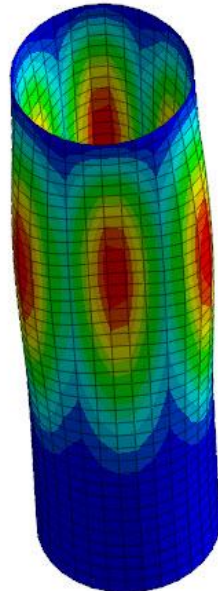
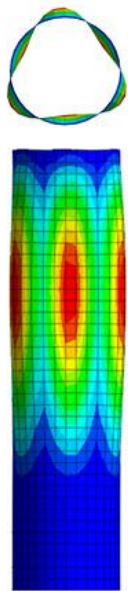
The most often used is Finite Element Method.



[23]



[24]



[22]



# Plate and shell structures

Asst. Prof. **Edita Papa Dukić**  
[edita.papa@uniri.hr](mailto:edita.papa@uniri.hr)  
G-329



Asst. Prof. **Nina Čeh**  
[nina.ceh@uniri.hr](mailto:nina.ceh@uniri.hr)  
G-332



# Plate and shell structures

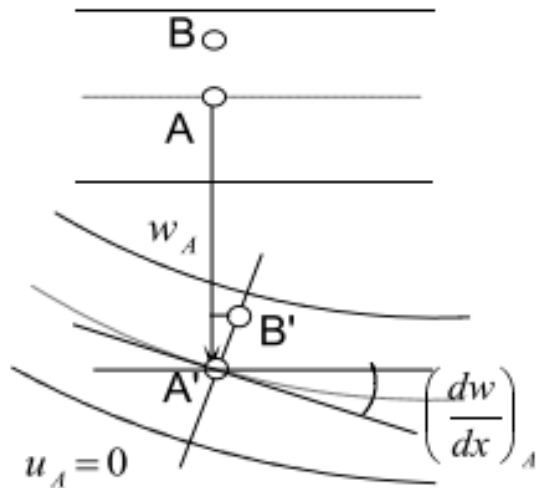


Different types of plate and shell structures in Civil Engineering: **walls, membranes, plates and shells.**

# Plate and shell structures

What is the difference between **beams** and **plates**?

## Beam structures

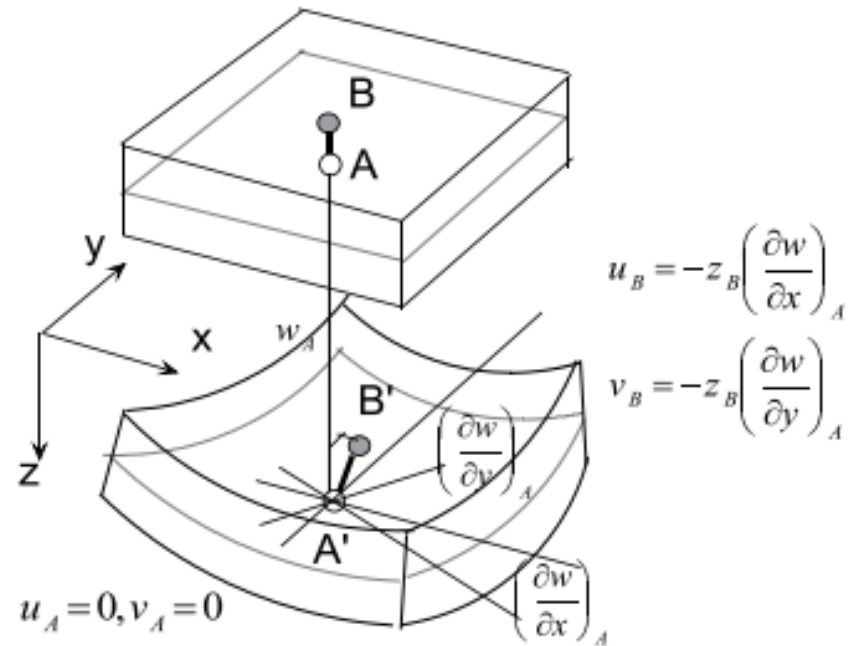


$$w_B = w_A$$

$$u_B = -z \left( \frac{dw}{dx} \right)_A$$

Displacements of all points are determined by the neutral axis displacement

## Plate structures



Displacements of all points are determined by the neutral **plane** displacement

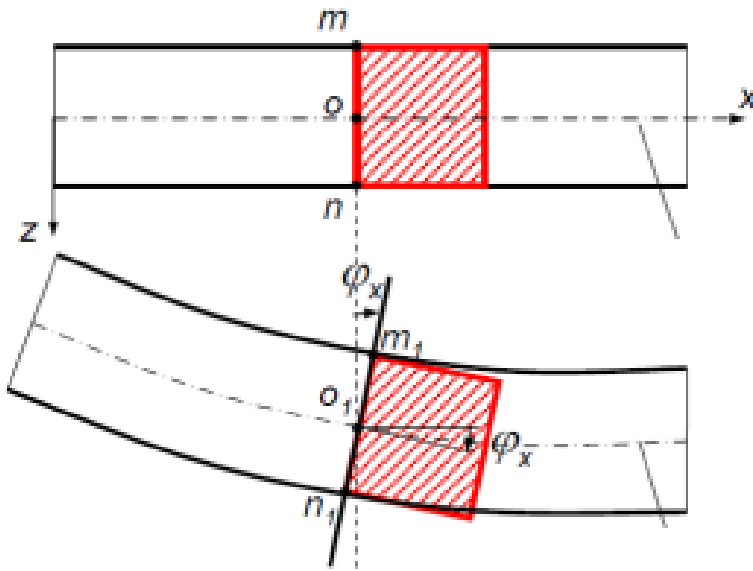
# Plate and shell structures

What is the difference between **thin** and **thick plates**?

**Bernouli I + II: Kirchhoff**

$$\varphi_x = \frac{\partial W}{\partial x} \quad \varphi_y = \frac{\partial W}{\partial y}$$

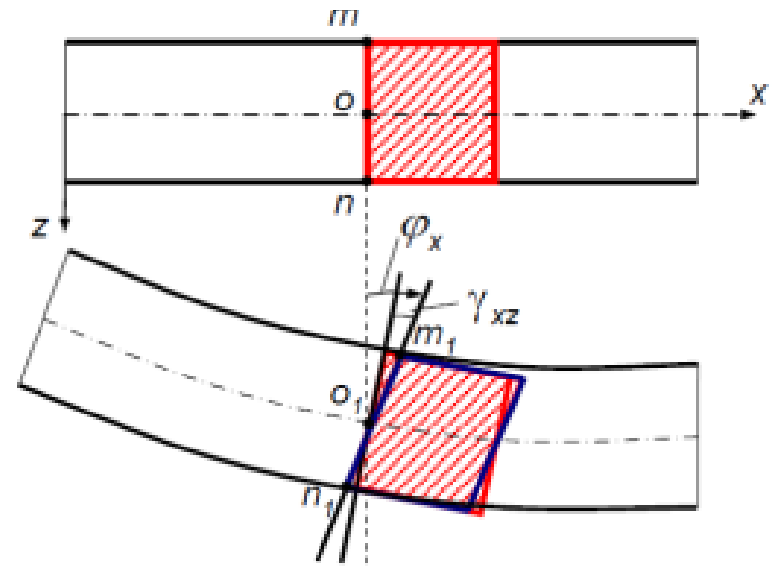
Shear strain is neglected



**Bernouli I: Reissner-Mindlin**

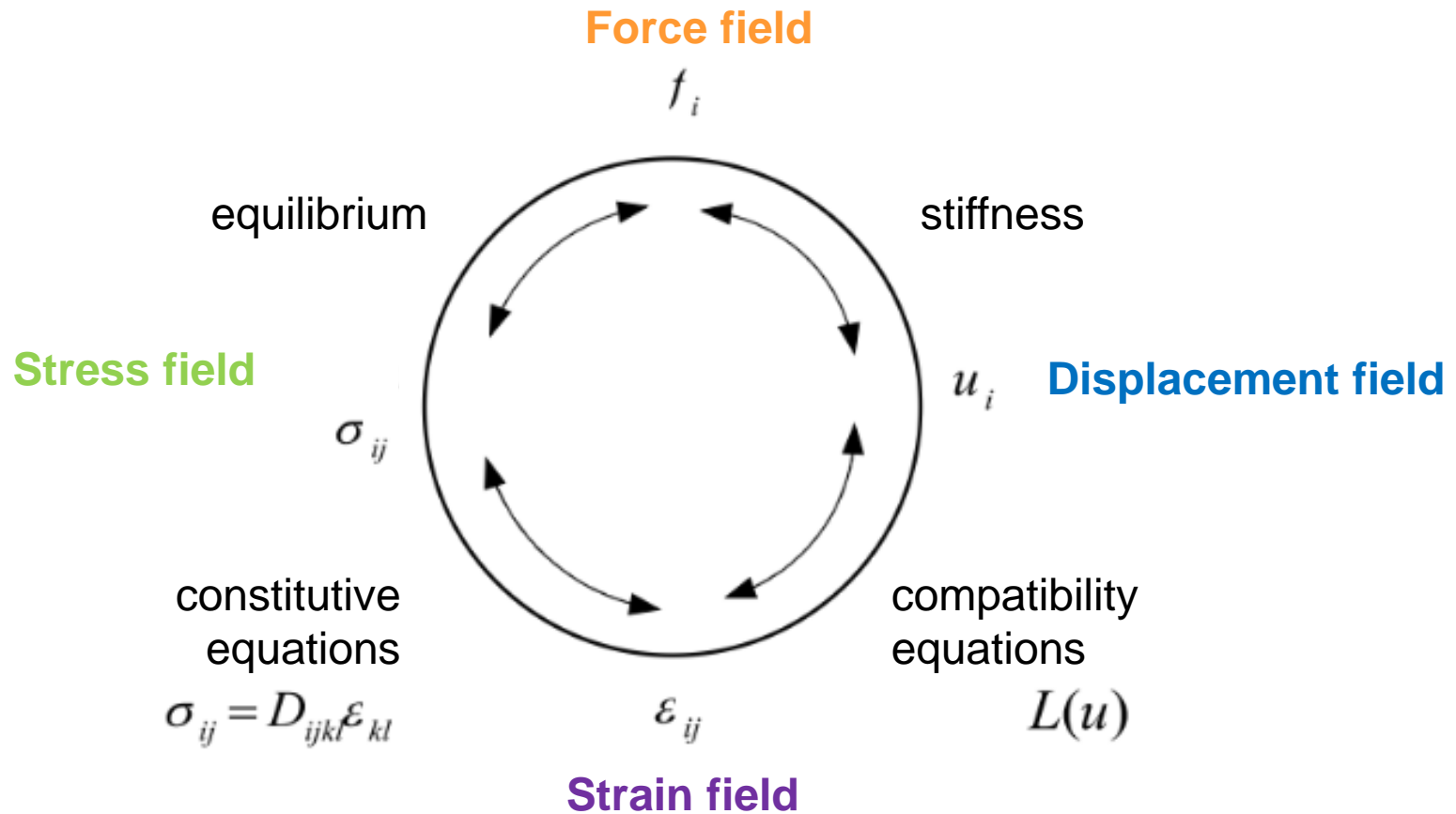
$$\varphi_x = \frac{\partial W}{\partial x} + \gamma_{xz} \quad \varphi_y = \frac{\partial W}{\partial y} + \gamma_{yz}$$

Shear strain is **NOT** neglected



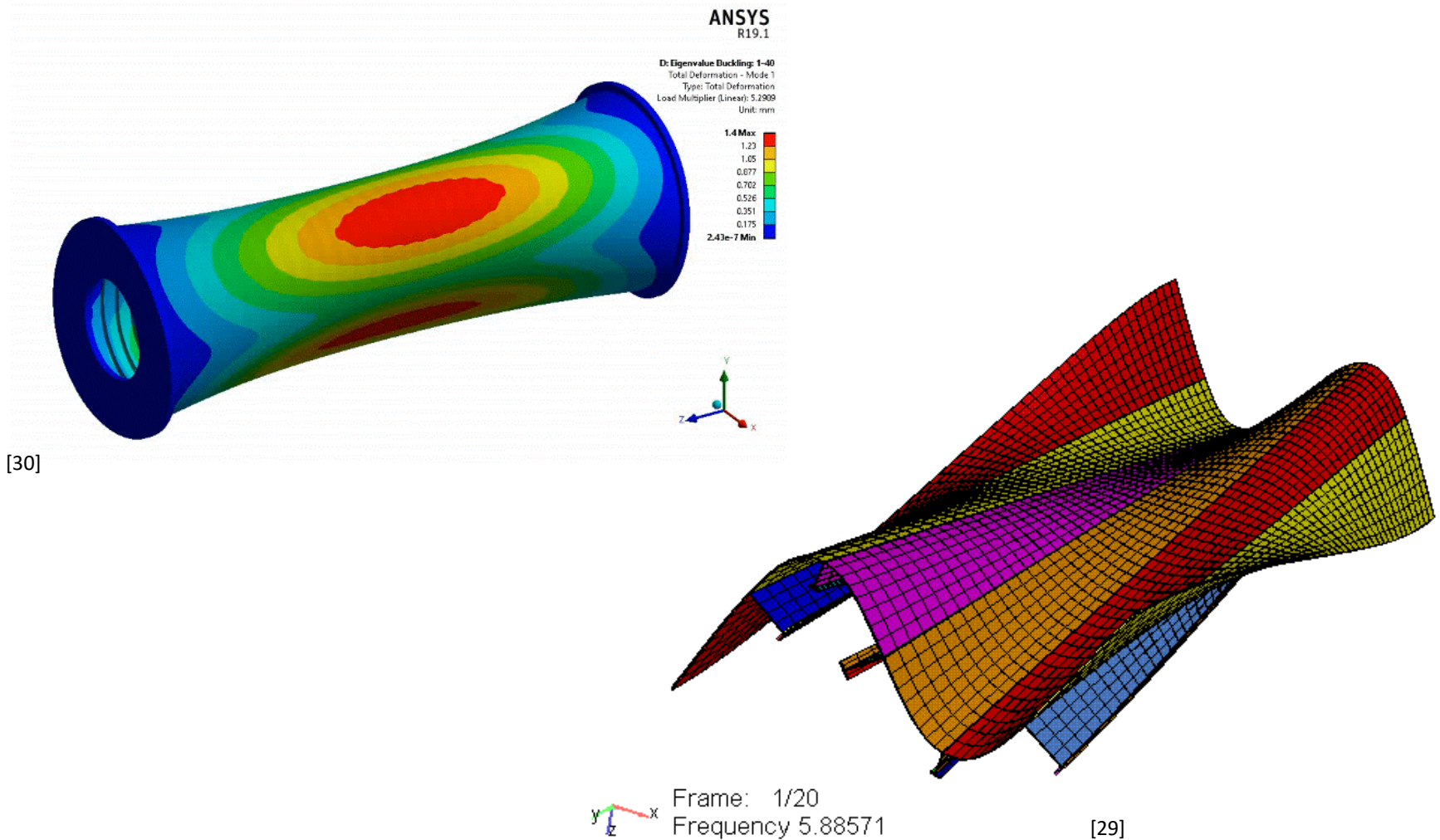
# Plate and shell structures

How are **forces**, **displacements**, **strains** and **stresses** related?



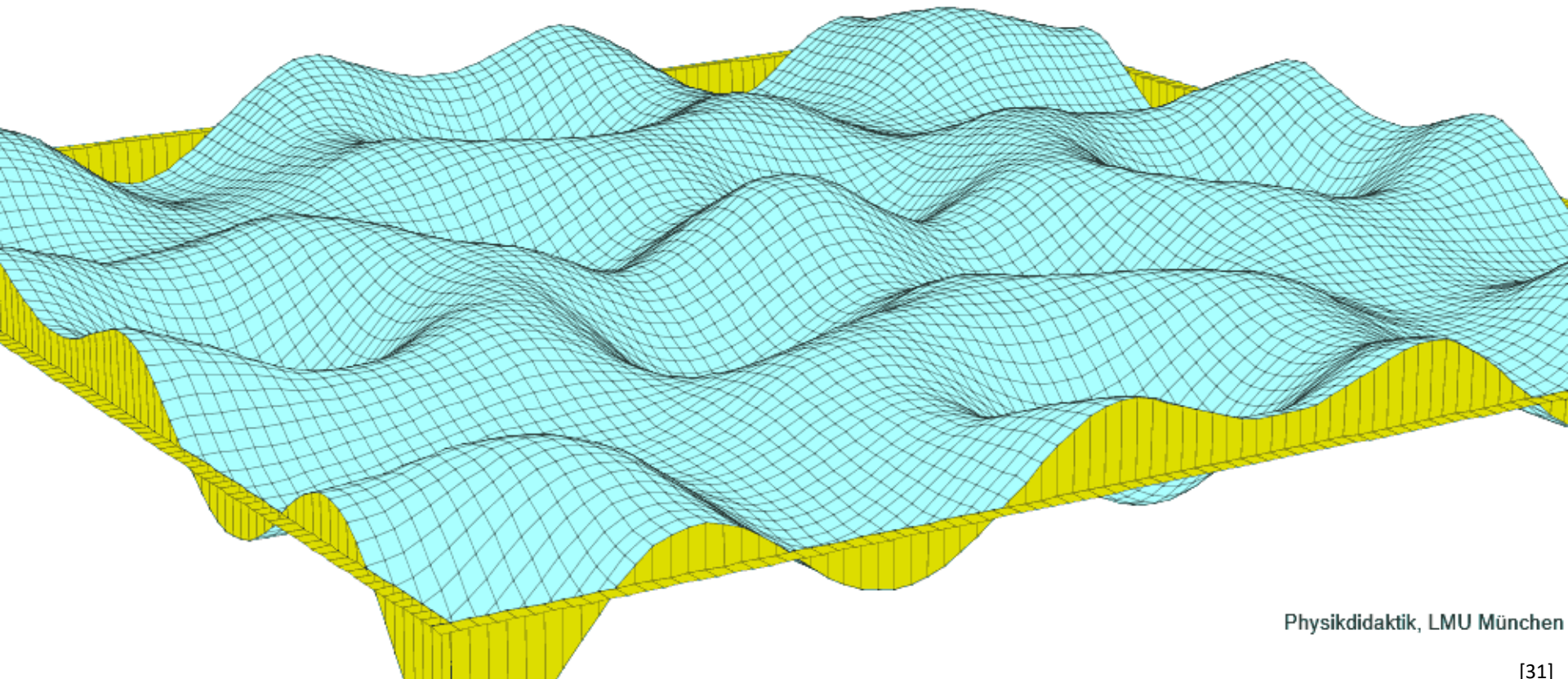
# Plate and shell structures

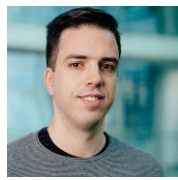
Introduction to numerical methods for static and dynamic analysis of plate structures – **finite difference method, Rayleigh-Ritz method and the finite element method.**



# Plate and shell structures

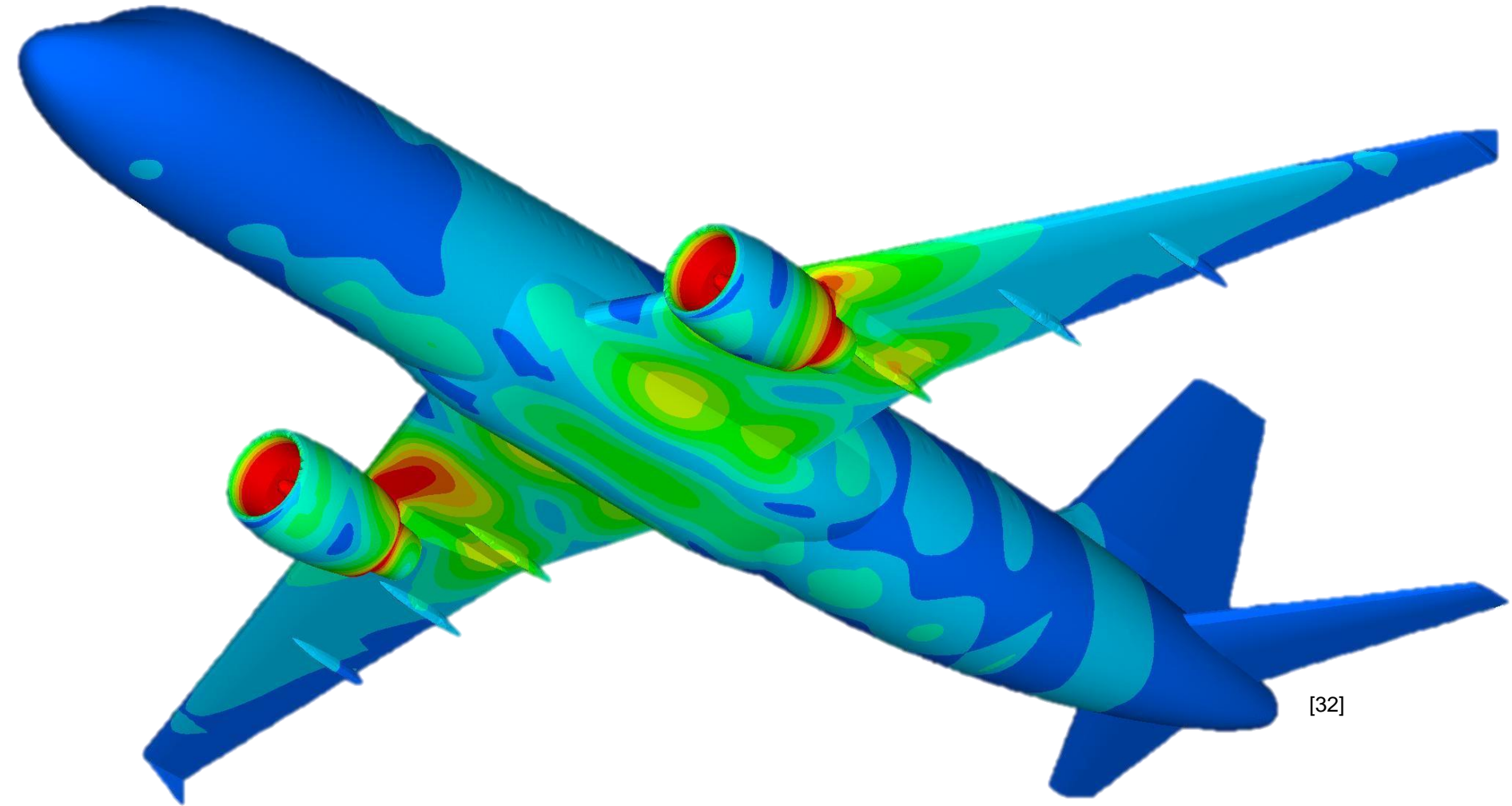
Introduction to numerical methods for static and dynamic analysis of plate structures – **finite difference method, Rayleigh-Ritz method and the finite element method.**





Asst. Prof. **Teo Mudrić**  
[teo.mudric2@uniri.hr](mailto:teo.mudric2@uniri.hr)  
G-333

# Energy Methods in Applied Mechanics



[32]



# Energy Methods in Applied Mechanics

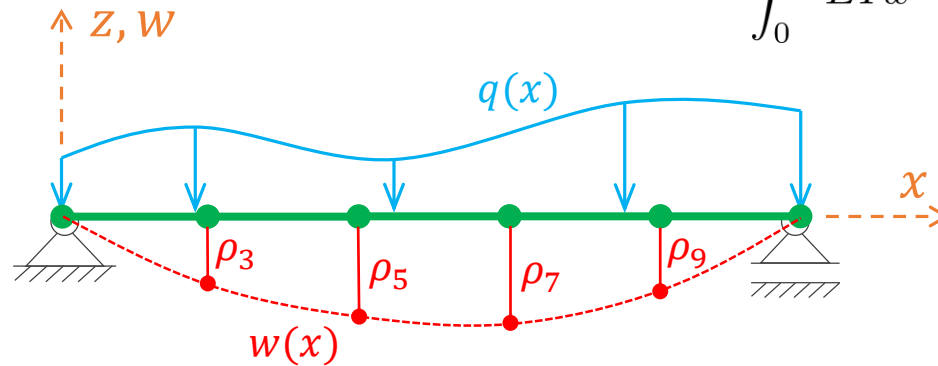
Strong form of the governing equation

$$EIw^{IV} = q$$

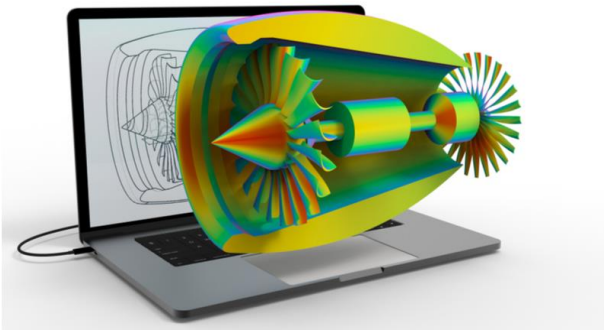
$$\begin{aligned} w(0) &= 0 \\ w(L) &= 0 \\ w''(0) &= 0 \\ w''(L) &= 0 \end{aligned}$$

Weak form of the governing equation

$$\int_0^L EIw''\bar{w}'' dx - \int_0^L q\bar{w}'' dx = 0$$



Practical for computer implementation  
(Finite element method)



[33]

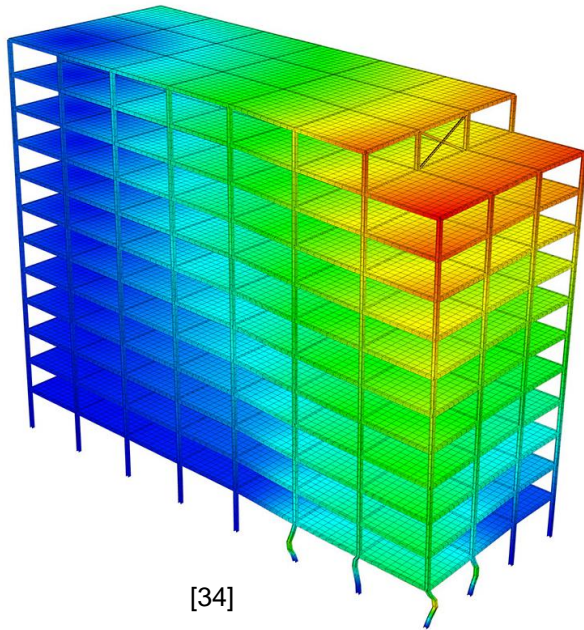
Finite element method  
approximation of the  
governing equation

$$\begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix}$$

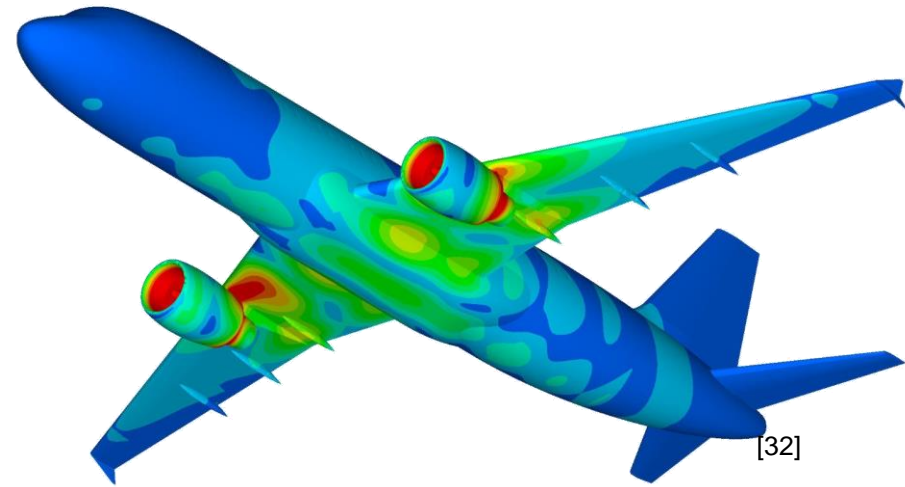
$$\mathbf{K}\boldsymbol{\rho} = \mathbf{F} \rightarrow \boldsymbol{\rho} = \mathbf{K}^{-1}\mathbf{F}$$

Procedures for approximate solutions satisfying the weak form will be described, with emphasis on the Finite element method

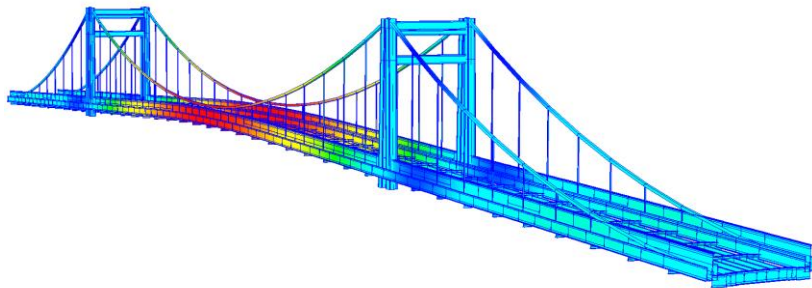
# Energy Methods in Applied Mechanics



[34]



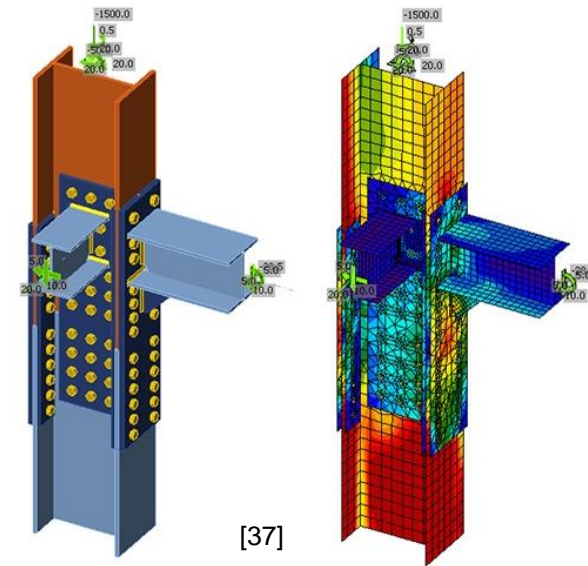
[32]



[35]



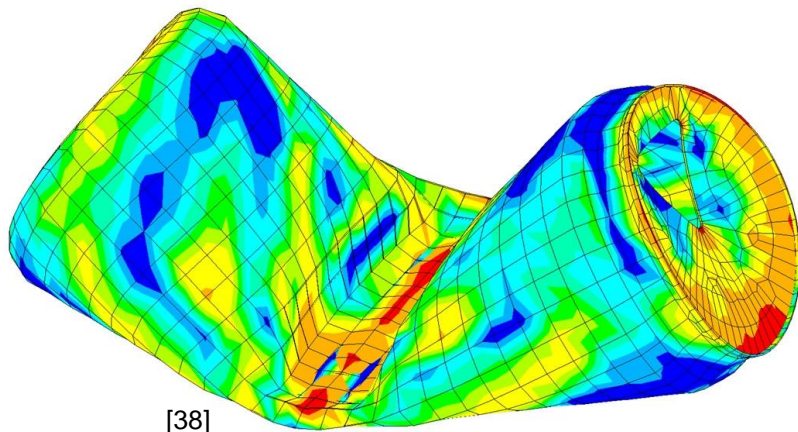
[36]



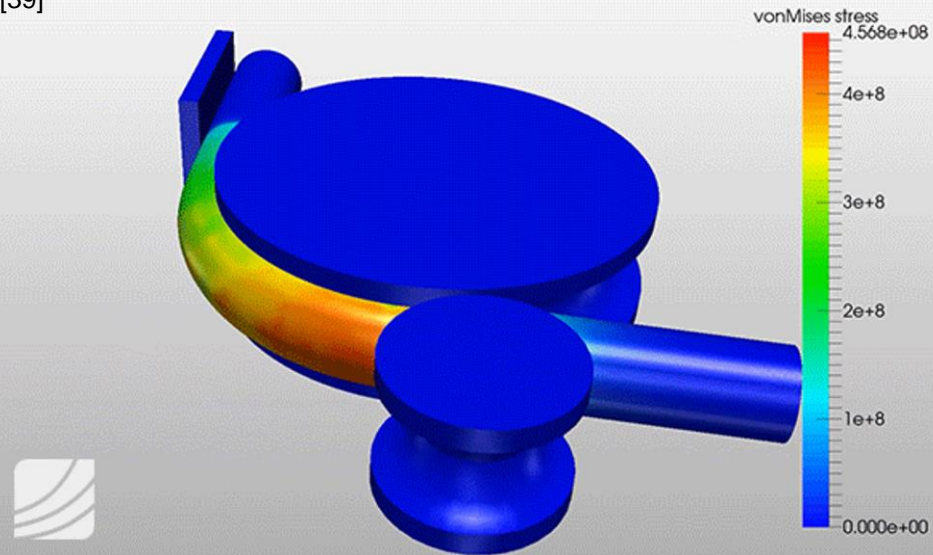
[37]

**A wide variety of complex mechanical problems can be analysed with software implementing the Finite element method**

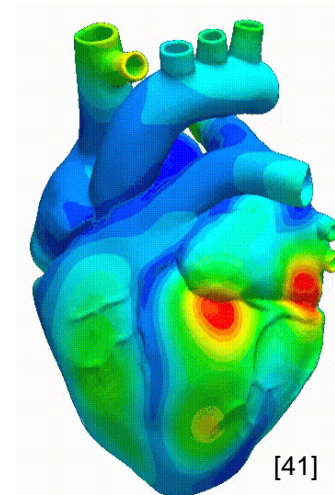
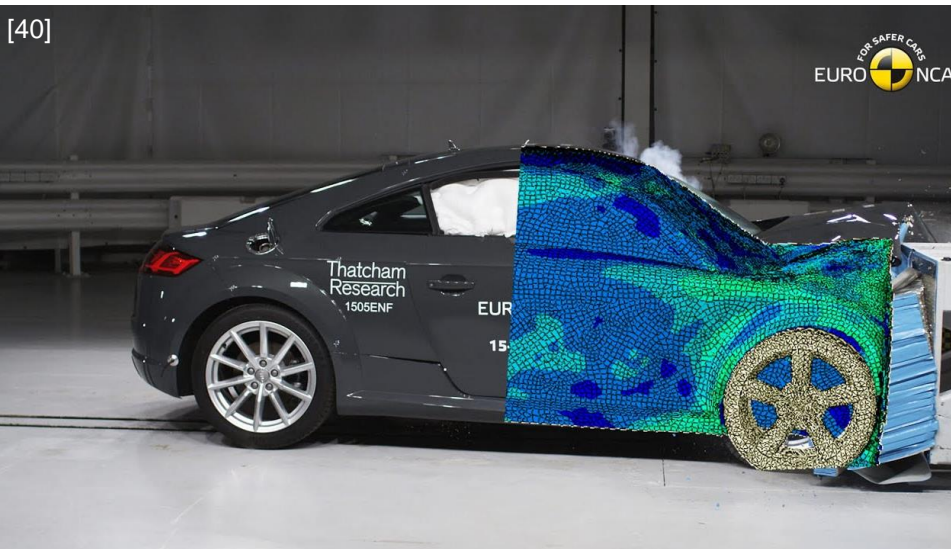
# Energy Methods in Applied Mechanics



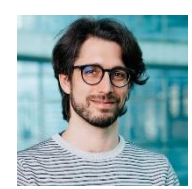
[39]



[40]



A wide variety of complex mechanical problems can be analysed with software implementing the Finite element method



Assoc. Prof. **Leo Škec**

[leo.skec@uniri.hr](mailto:leo.skec@uniri.hr)

G-330



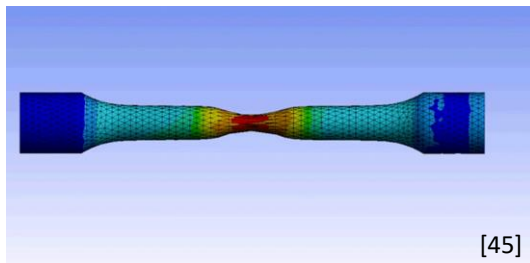
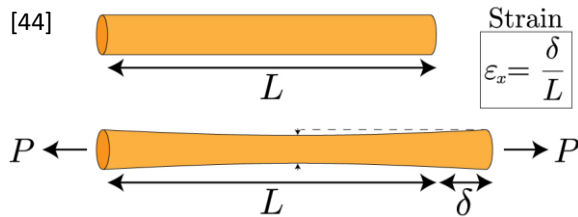
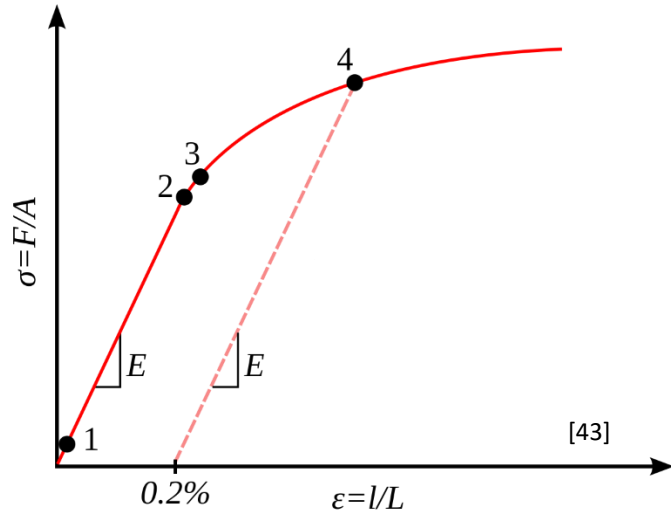
[42]

# Introduction to plasticity and damage modelling

# Introduction to plasticity and damage modelling

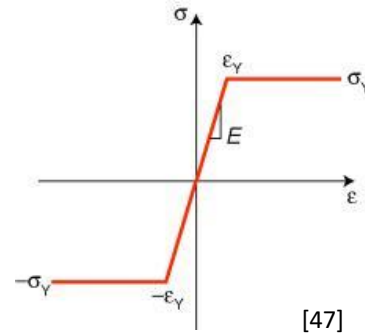
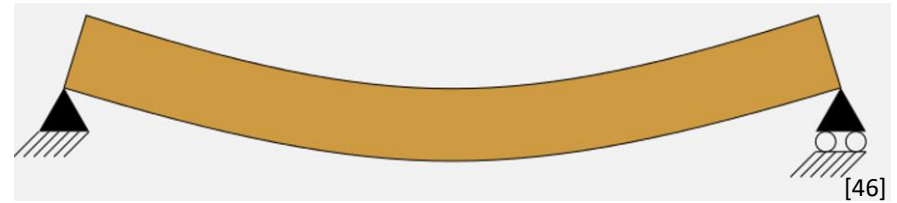
## UNIAXIAL STRESS STATE

- Plastic deformations

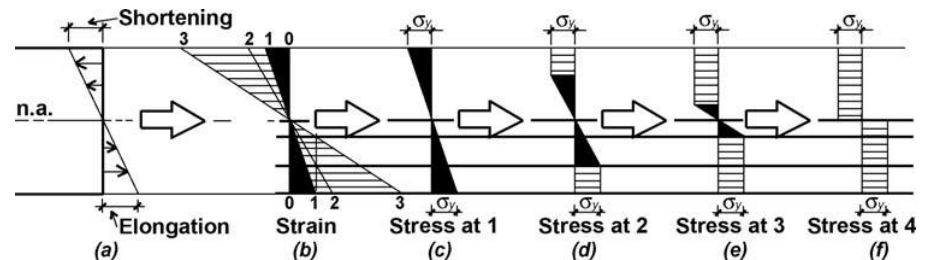
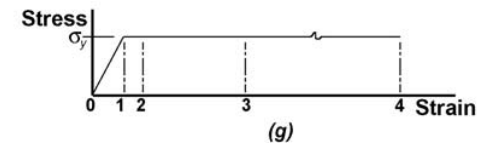


## ELASTO-PLASTIC BENDING

- cross-section gradual plastification and plastic joint development



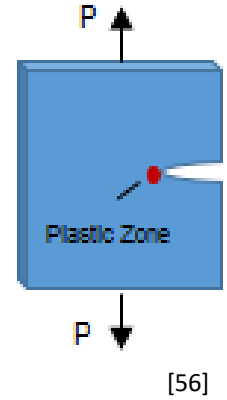
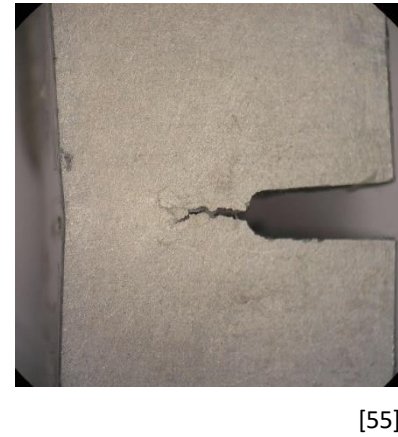
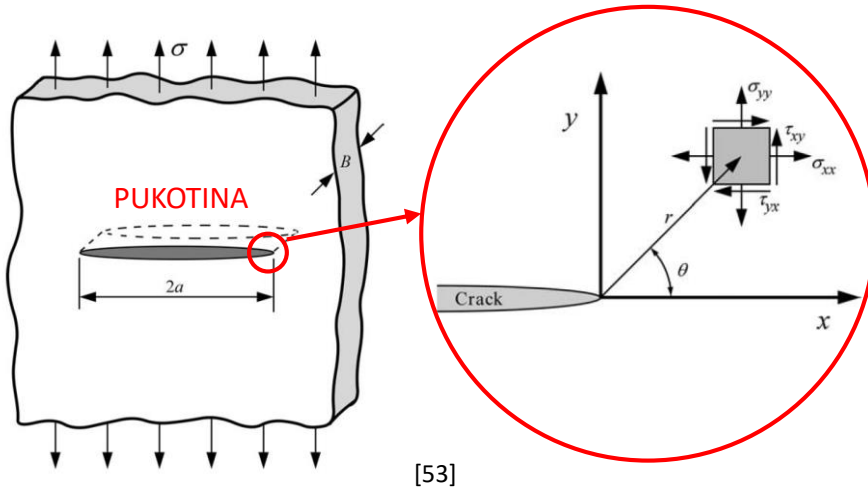
## Prandtl model





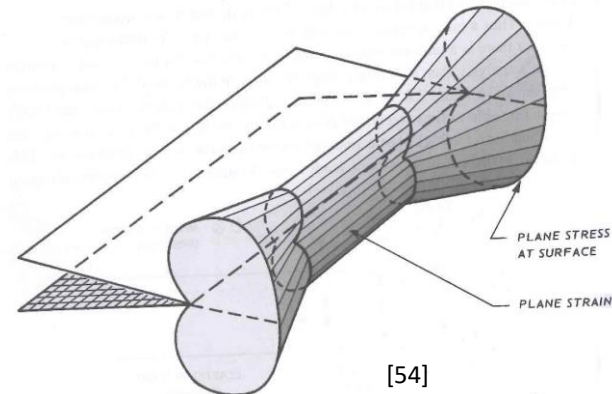
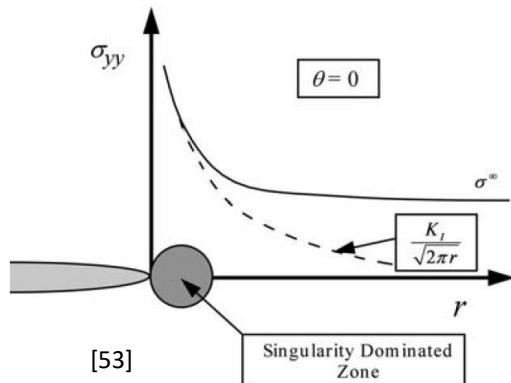
# Introduction to plasticity and damage modelling

**FRACTURE MECHANICS** – when and why does a crack develop in a material?



**Linear elastic fracture mechanics** – brittle material fracture

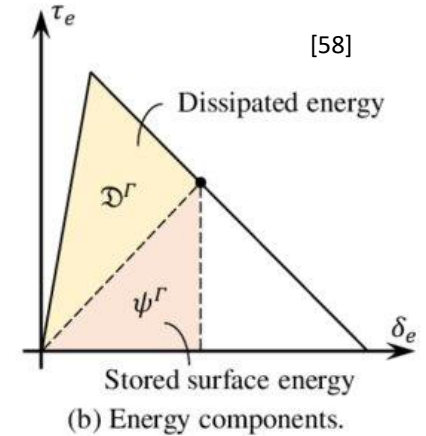
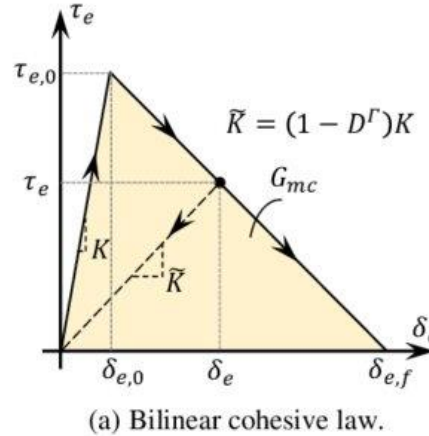
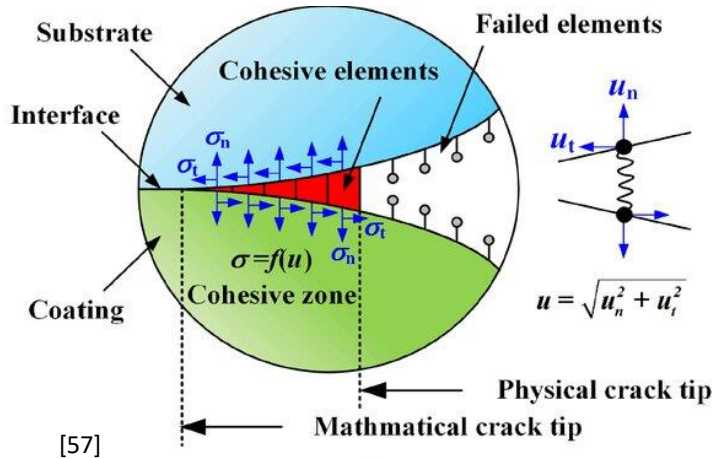
**Ductile fracture of the material** – a plastic zone (damage) forms behind the crack



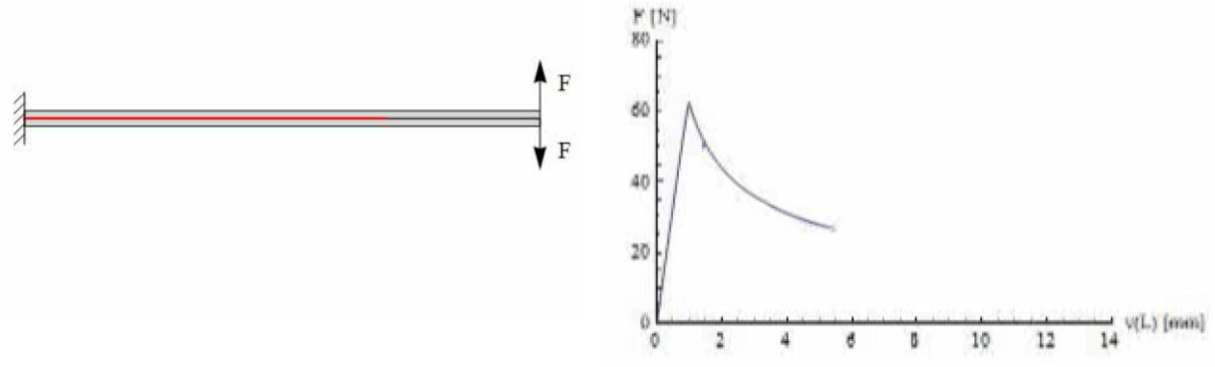
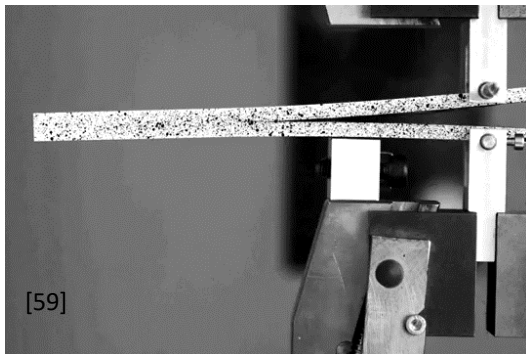
# Introduction to plasticity and damage modelling

## DAMAGE MECHANICS

- Models of progressive damage (softening) leading to material failure



Experimental determination of parameters that describe the fracture resistance of materials





# APPLIED MECHANICS

## IN MODERN ENGINEERING PRACTICE

Detailed description of the courses can be found on [www.gradri.hr](http://www.gradri.hr).  
<https://gradri.uniri.hr/en/lifelong-education/>

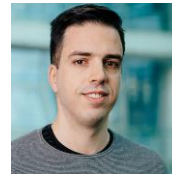
Contacts for additional information about the courses:



Asst. Prof. **Edita Papa Dukić**  
[edita.papa@uniri.hr](mailto:edita.papa@uniri.hr)  
G-329



Asst. Prof. **Nina Čeh**  
[nina.ceh@uniri.hr](mailto:nina.ceh@uniri.hr)  
G-332



Asst. Prof. **Teo Mudrić**  
[teo.mudric2@uniri.hr](mailto:teo.mudric2@uniri.hr)  
G-333



Assoc. Prof. **Leo Škec**  
[leo.skec@uniri.hr](mailto:leo.skec@uniri.hr)  
G-330

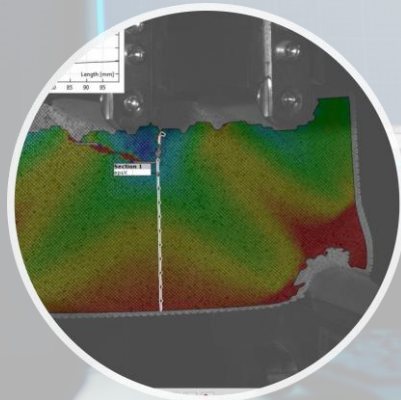
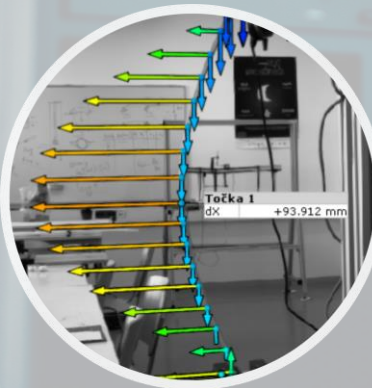
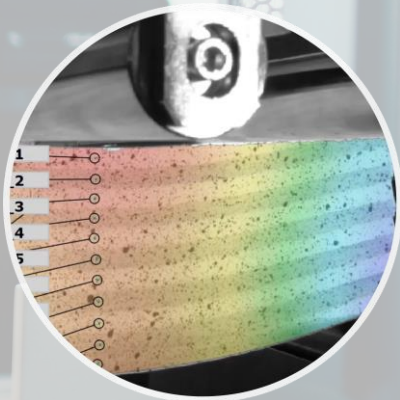


Assoc. Prof. **Dragan Ribarić**  
[dragan.ribaric@uniri.hr](mailto:dragan.ribaric@uniri.hr)  
G-328

**REMARK:** As part of the graduate study program it is possible to choose only part of the Minor courses, but then the Minor cannot be acquired.

# APPLIED MECHANICS

## IN MODERN ENGINEERING PRACTICE



G

F

Sveučilište  
u Rijeci  
Građevinski  
fakultet

# References:

- <http://brookreport.com/best-colleges-for-civil-engineering-majors-2022/>
- <https://veryrealandrew.com/site-visits/2019/12/16>
- <https://www.utilitydesign.co.uk/blog/celebration-modern-architecture-styles/>
- <https://edition.cnn.com/style/article/famous-buildings-dubai/index.html>
- <https://www.grandtouringautos.com/model/rimac-nevera/>
- [https://en.wikipedia.org/wiki/Differential\\_%28mechanical\\_device%29](https://en.wikipedia.org/wiki/Differential_%28mechanical_device%29)
- [https://mlrit.ac.in/wp-content/uploads/2022/03/istock-541144900\\_0.jpeg](https://mlrit.ac.in/wp-content/uploads/2022/03/istock-541144900_0.jpeg)
- <https://www.automotiveplastics.com/automotive-plastics-today/chassis/>
- <https://images.unsplash.com/photo-1559023234-1e773470544f?ixlib=rb-4.0.3&ixid=M3wxMjA3fDB8MHxzZWZFY2h8Mnx8cGxhbmUIMjB0YWtlb2ZmfGVufDB8fDB8fHww&w=1000&q=80>
- <https://www.skyfilabs.com/blog/list-of-major-design-projects-for-mechanical-engineering-students>
- <https://csb-scb.com/>
- <https://centralgaheart.com/need-know-heart-stent/>
- <https://www.perfectbracesacademy.com/uploads/products/t/l-p-91-p-24-4tnhw.png>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://www.pinterest.com/>
- <https://seele.com/references/strasbourg-railway-station>
- <https://www.civilprojectsonline.com/tag/lotus-temple-as-a-shell-structure/>
- <https://www.novilist.hr/rijeka-regija/rijeka/oporavak-stizu-prve-naznake-boljih-vremena-luka-rijeka-u-sedam-mjeseci-povecala-promet/>
- <https://pdh-pro.com/course/materials-for-embankment-dams/>
- <https://www.sintef.no/projectweb/isogeometric-analysis/collaborators/projects/icada/results/>
- [Linear elastic buckling of ring-stiffened cylindrical shell  
\(https://www.finglowconsultants.co.uk/2023-fcl-busy-week\)](https://www.finglowconsultants.co.uk/2023-fcl-busy-week)
- <https://www.en.didaktik.physik.uni-muenchen.de/multimedia/waves/animations/index.html>
- <https://www.stressebook.com/finite-element-analysis-in-a-nut-shell/>
- <https://www.imeche.org/news/news-article/modern-finite-element-analysis-achieves-accurate-results-with-little-effort>
- <https://aecmag.com/simulation/msc-software-partners-with-ingeciber-to-boost-simulation/>
- <https://www.midasbridge.com/en/project-application/three-sisters-bridges>
- [https://www.researchgate.net/figure/The-FE-model-of-the-studied-wind-turbine\\_fig5\\_277009482](https://www.researchgate.net/figure/The-FE-model-of-the-studied-wind-turbine_fig5_277009482)
- <https://www.h-conner.co.th/Article/Detail/151373>
- <https://www.linkedin.com/pulse/shells-vs-solids-finite-element-analysis-quick-review-kuusisto-p-e->
- [https://commons.wikimedia.org/wiki/File:Bending\\_Analysis\\_of\\_an\\_Aluminium\\_Pipe.gif](https://commons.wikimedia.org/wiki/File:Bending_Analysis_of_an_Aluminium_Pipe.gif)
- <https://i.ytimg.com/vi/lol15USYnDw/maxresdefault.jpg>
- <https://www.cies.unsw.edu.au/scaled-boundary-finite-element-method-2a>
- <https://www.foxnews.com/us/north-carolina-official-says-completely-severed-crack-roller-coaster-was-visible-days-closing>
- [https://en.wikipedia.org/wiki/Plasticity\\_\(physics\)](https://en.wikipedia.org/wiki/Plasticity_(physics))
- <https://www.bu.edu/moss/mechanics-of-materials-strain/>
- <https://yasincapar.com/>
- [https://benjdd.com/courses/cs110/spring-2019/pas/bridge\\_deflection/index.html](https://benjdd.com/courses/cs110/spring-2019/pas/bridge_deflection/index.html)
- [https://www.doitpoms.ac.uk/tlplib/beam\\_bending/plastic.php](https://www.doitpoms.ac.uk/tlplib/beam_bending/plastic.php)
- <https://jonochshorn.com/structuralelements/book/1.08-strength.html>
- [https://www.efunda.com/formulae/solid\\_mechanics/mat\\_mechanics/stress.cfm](https://www.efunda.com/formulae/solid_mechanics/mat_mechanics/stress.cfm)
- [https://en.wikipedia.org/wiki/Von\\_Mises\\_yield\\_criterion](https://en.wikipedia.org/wiki/Von_Mises_yield_criterion)
- [https://www.reddit.com/r/fea/comments/smm5x3/is\\_this\\_still\\_a\\_trend/](https://www.reddit.com/r/fea/comments/smm5x3/is_this_still_a_trend/)
- <https://skill-lync.com/student-projects/assignment-6-frontal-crash-simulation-challenge-51>
- [Ted L. Anderson, Fracture Mechanics: Fundamentals and Applications, Fourth Edition, CRC Press, 2017](https://www.ted.com/talks/ted_l_anderson_fracture_mechanics_fundamentals_and_applications)
- <https://pdfs.semanticscholar.org/42f2/b3e97ce3f8bcaffacd808c8209dfe6e7e0f7.pdf>
- <https://www.metallurgyfordummies.com/tag/fracture-mechanics-concepts.html>
- <https://mechanicalc.com/reference/fracture-mechanics>
- [https://www.researchgate.net/publication/352136601\\_Monitoring\\_of\\_Crack\\_Initiation\\_at\\_CoatingSubstrate\\_Interface\\_by\\_Residual\\_Magnetic\\_Field\\_Measurement/figure?lo=1](https://www.researchgate.net/publication/352136601_Monitoring_of_Crack_Initiation_at_CoatingSubstrate_Interface_by_Residual_Magnetic_Field_Measurement/figure?lo=1)
- [https://www.researchgate.net/publication/334303096\\_Adaptive\\_Discrete-Smeared\\_Crack\\_A-DISC\\_Model\\_for\\_Multi-Scale\\_Progressive\\_Damage\\_in\\_Composites/figures?lo=1](https://www.researchgate.net/publication/334303096_Adaptive_Discrete-Smeared_Crack_A-DISC_Model_for_Multi-Scale_Progressive_Damage_in_Composites/figures?lo=1)
- <https://www.youtube.com/watch?v=ssy9ReoC46Y>
- <https://www.birdair.com/technology/tensile-architecture/aesthetics-design/>